ECS455 Chapter 2

Cellular Systems

2.2 Co-Channel Interference

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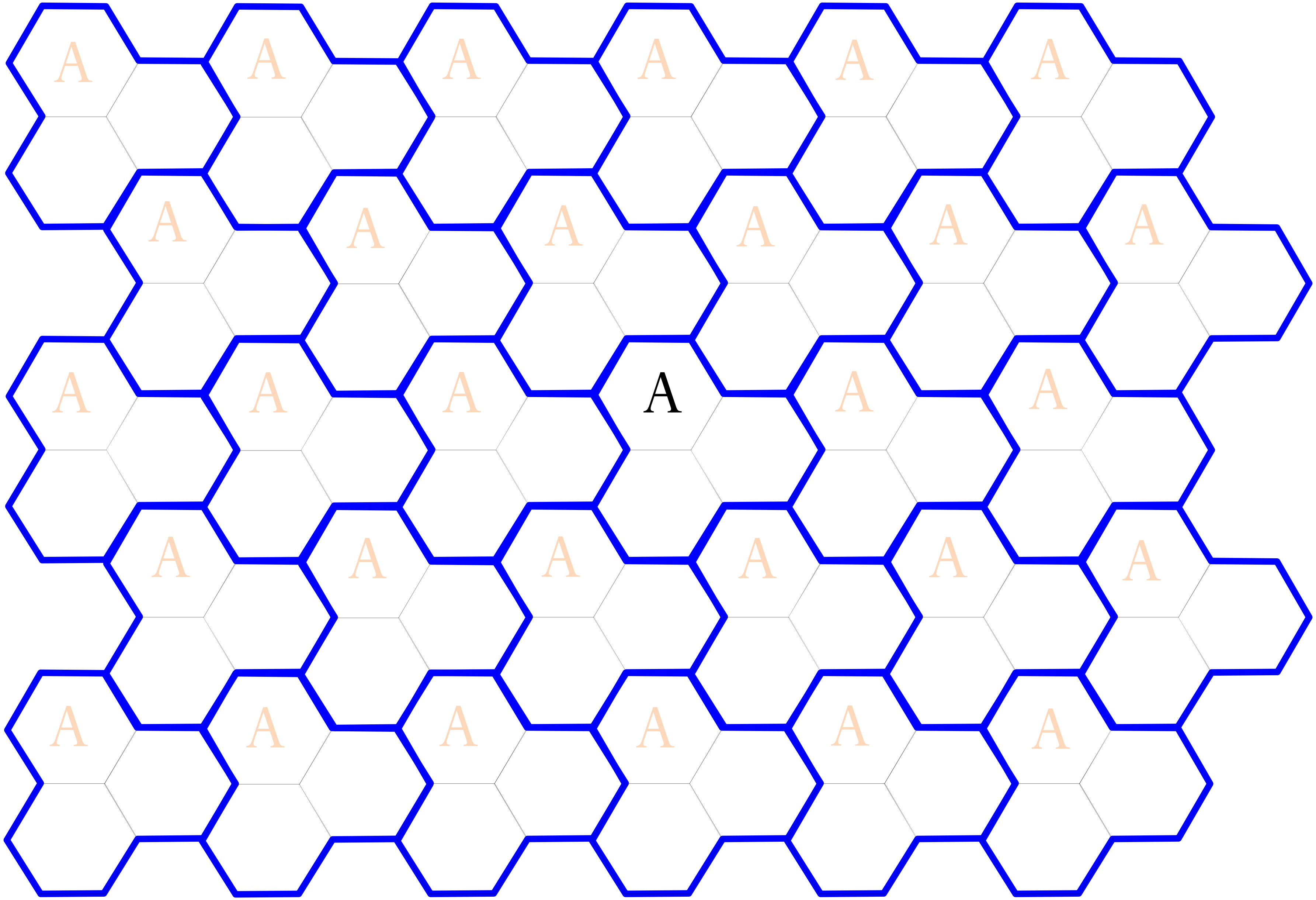
Office Hours:
BKD 3601-7

Tuesday	**9:30-10:30**
Tuesday	**13:30-14:30**
Thursday	**13:30-14:30**

Co-Channel Cells: Ex. N = 3



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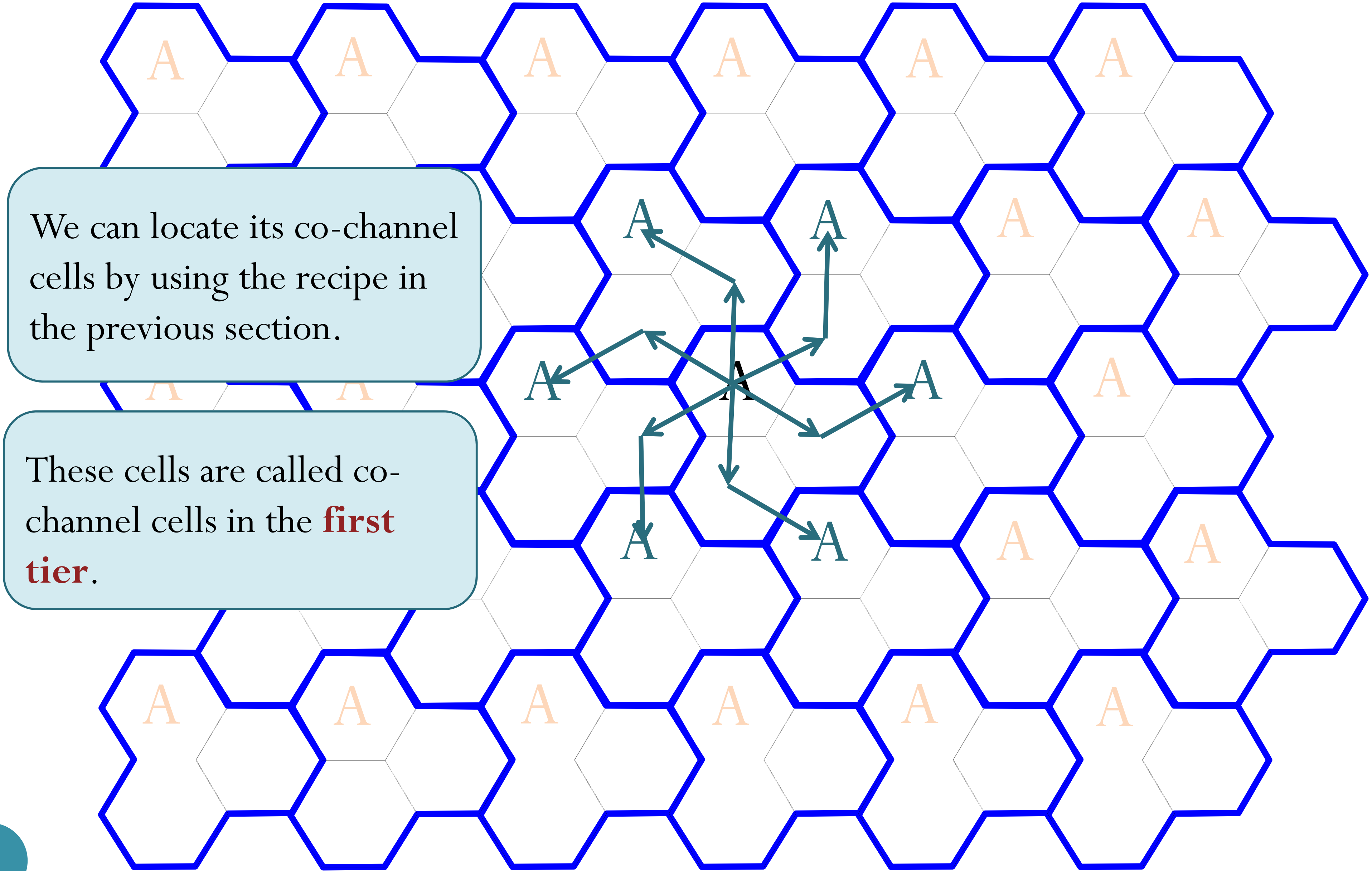
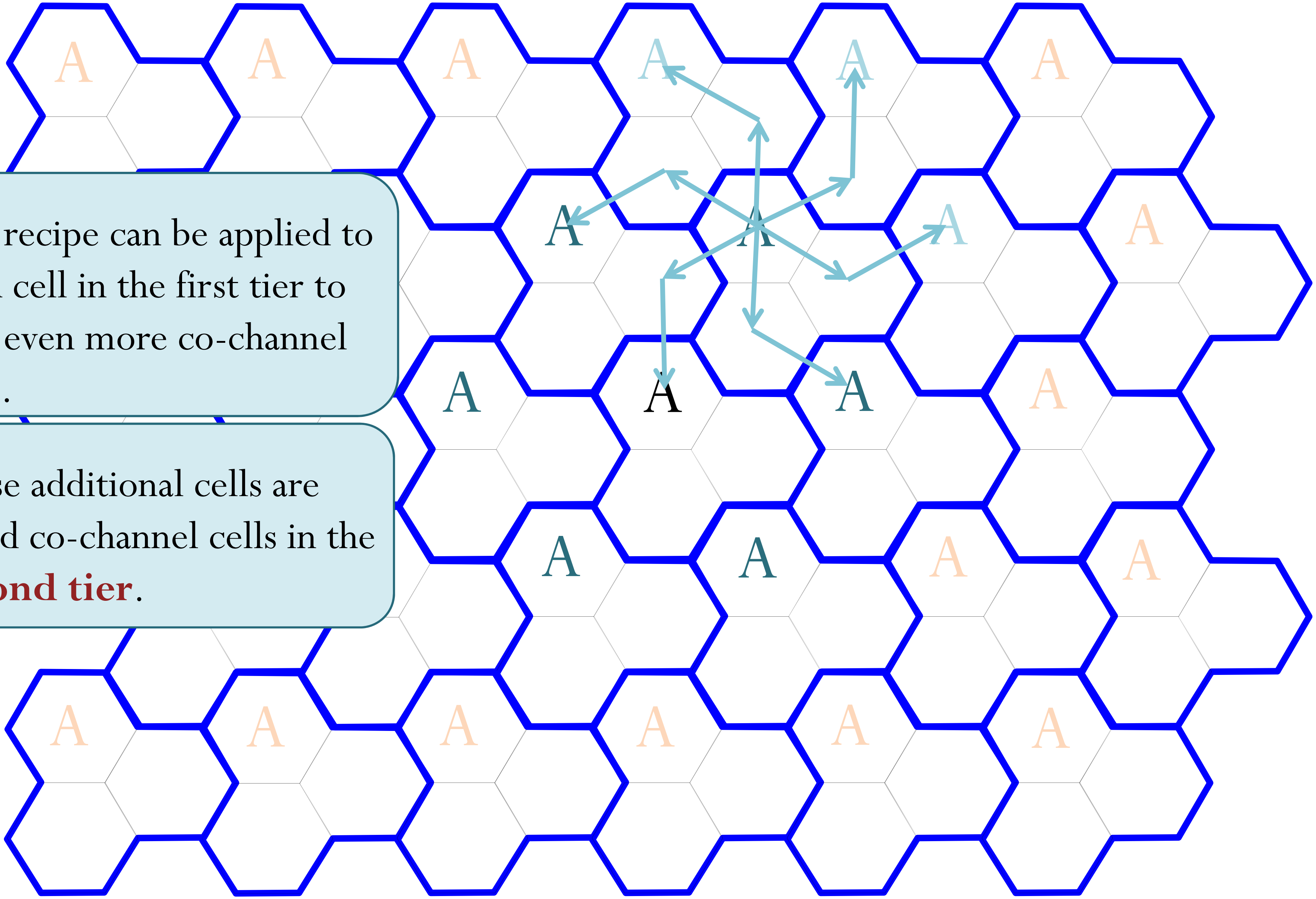


We can locate its co-channel cells by using the recipe in the previous section.

These cells are called co-channel cells in the **first tier**.

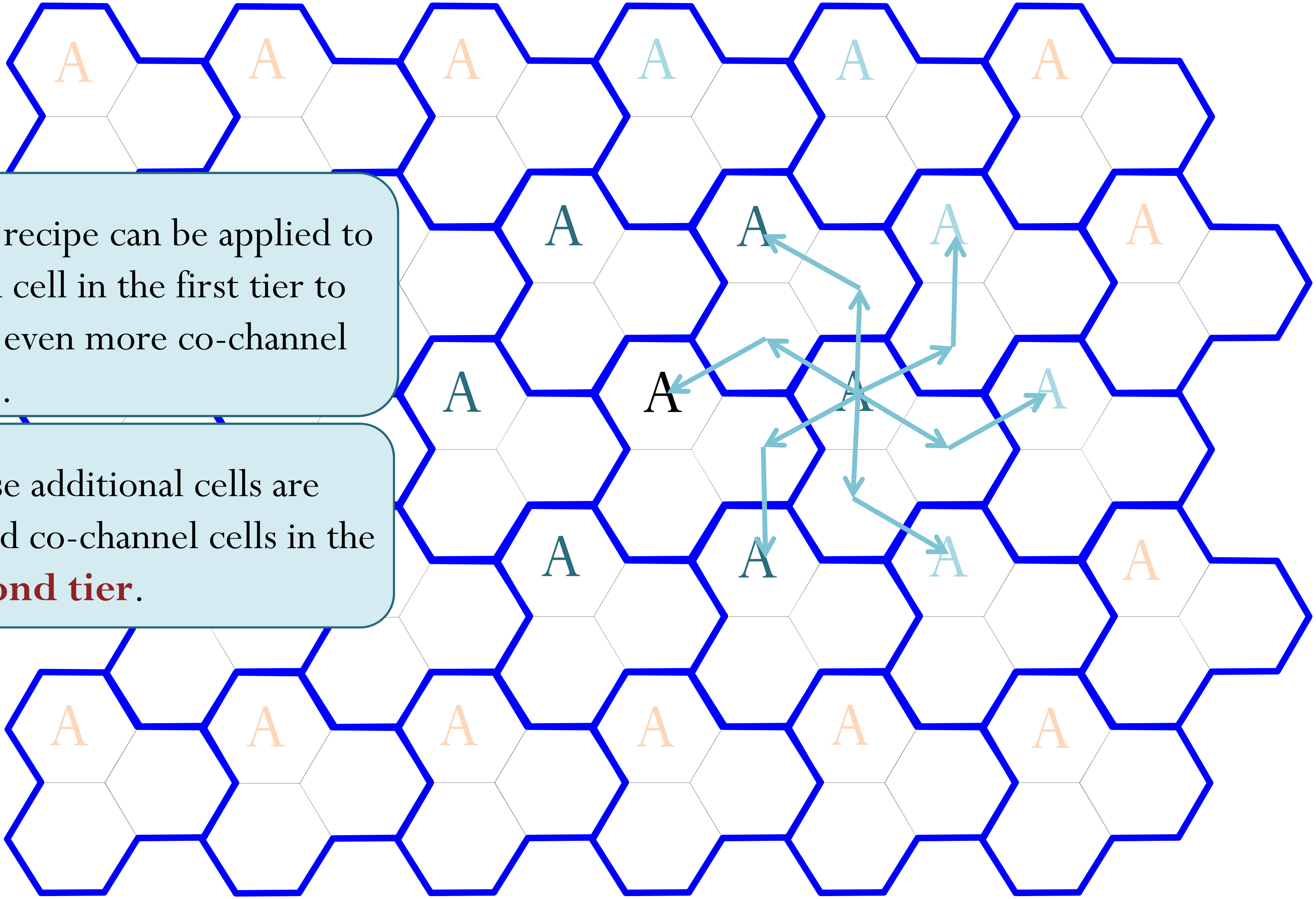
Co-Channel Cells: Ex. N = 3



The recipe can be applied to each cell in the first tier to find even more co-channel cells.

These additional cells are called co-channel cells in the **second tier**.

Co-Channel Cells: Ex. N = 3



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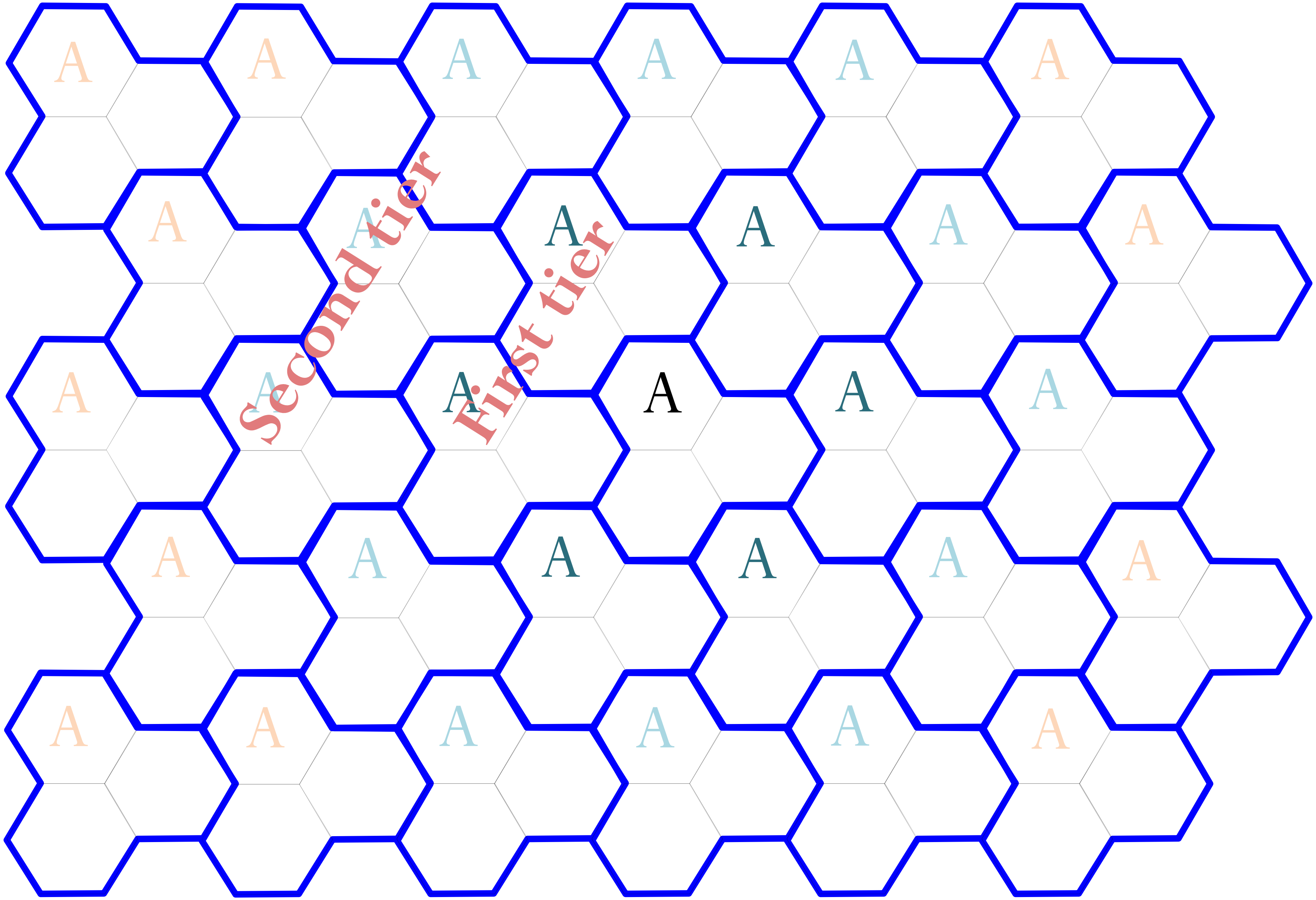

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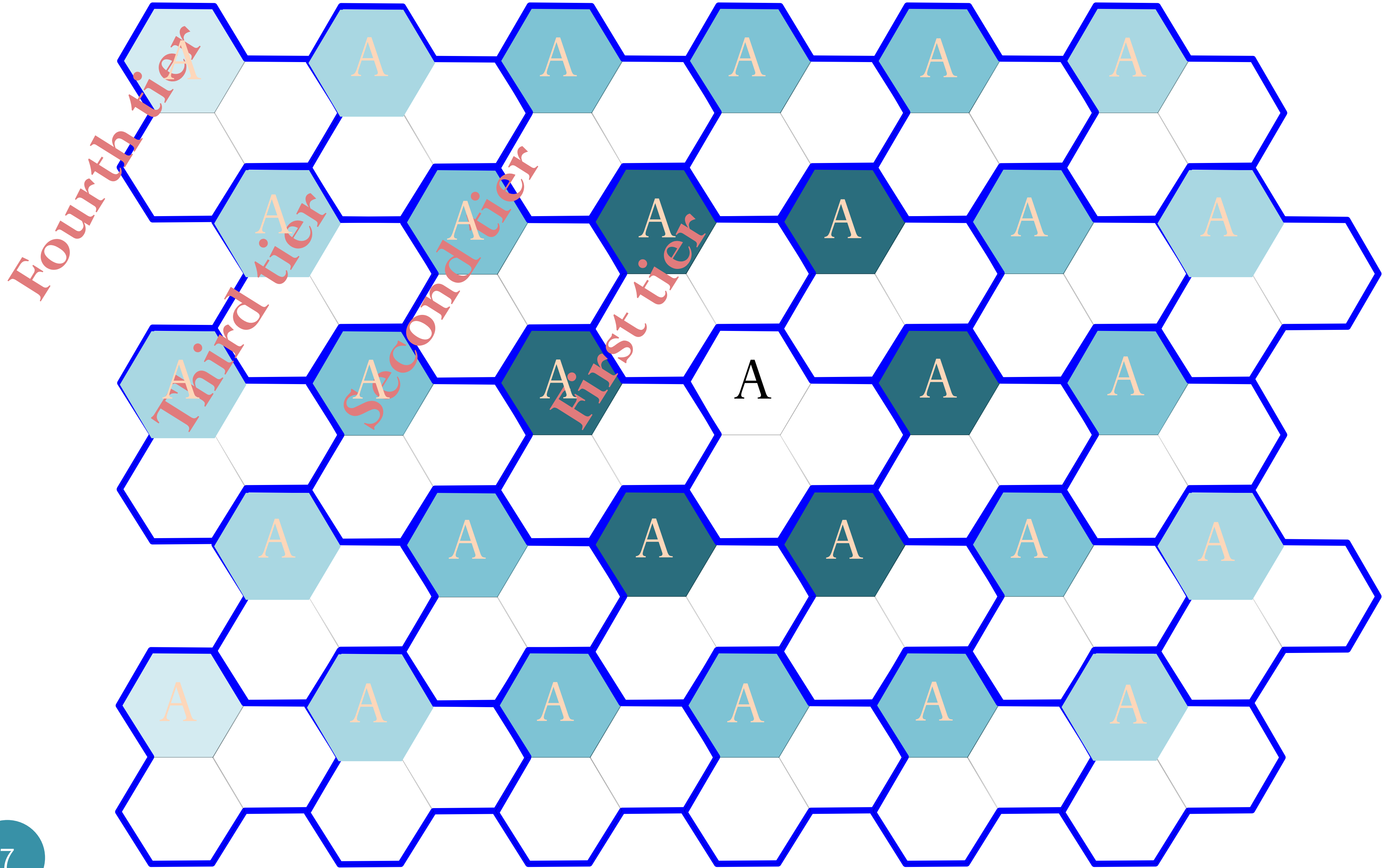
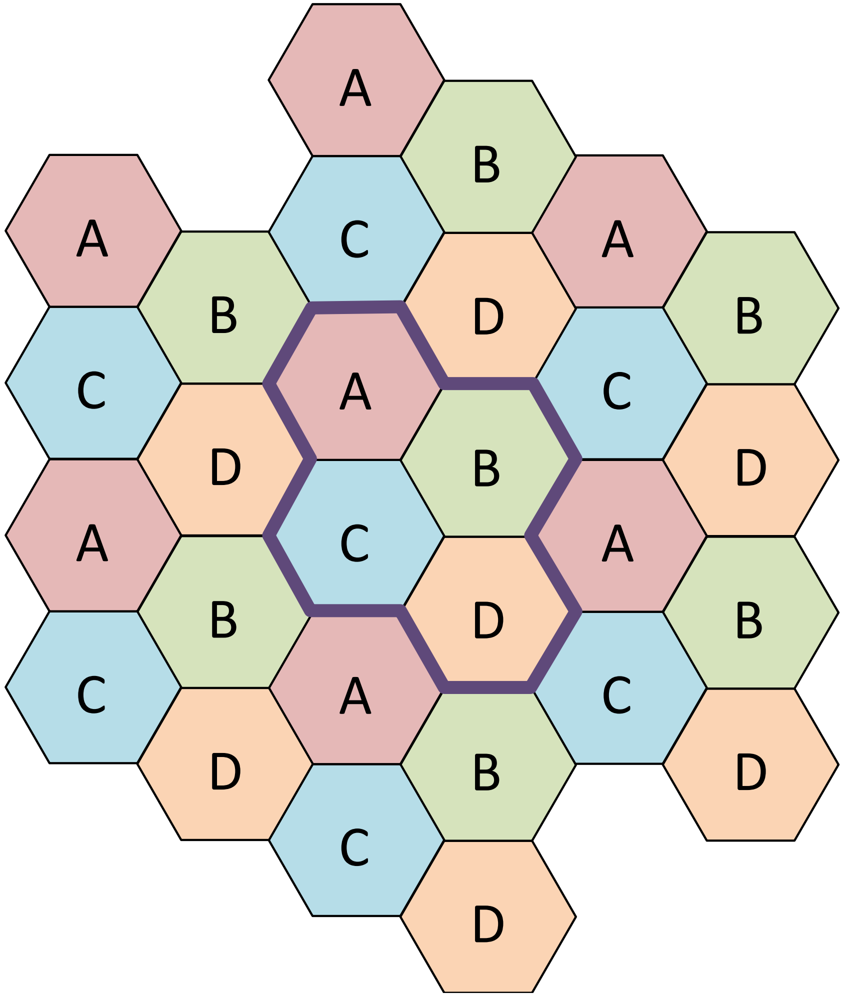


(Intercell)

Co-Channel Interference

- Frequency reuse → co-channel interference
- Consider only nearby interferers.
  - Power decreases rapidly as the distance increases.
- In a **fully equipped hexagonal-shaped** cellular system, there are always K = 6 cochannel interfering cells in the **first tier**.



Three Measures of Signal Quality

- For **noise-limited** systems, $\text{SNR} = \dfrac{P_r}{P_{\text{noise}}}$
- Consider both noise & interference: $\text{SINR} = \dfrac{P_r}{P_{\text{interference}} + P_{\text{noise}}}$
- The best cellular system design places users that share the same channel at a separation distance (as close as possible) where the intercell interference is just below the maximum tolerable level for the required data rate and BER.
- Good cellular system designs are **interference-limited**, meaning that the interference power is much larger than the noise power.

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}}$$

$P_{interference} \gg P_{noise}$

"Reliable" vs. "tolerable"?

(Why not as far as possible?)

Co-channel cells, must be spaced **far enough** apart so that interference between users in co-channel cells does not degrade **signal quality** below **tolerable** levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least **sixty times** higher than the noise/interference power.

[Klemens, 2010, p 54]

$$10\log_{10} 60 = 17.78 \approx 18 \text{ dB}$$

We will soon revisit and use these numbers for some more specific calculations

Review: Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma} \Rightarrow P_r = \frac{P_t K d_0^{\gamma}}{d^{\gamma}} \propto \frac{1}{d^{\gamma}}$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

$$P_r = \frac{k}{d^{\gamma}}$$

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.
- γ is the **path loss exponent**.
  - 2 in free-space model
  - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

[Goldsmith, 2005, Table 2.2]

SIR (S/I): Definition/Calculation

Caution: Not the same as the K shown on the previous slide

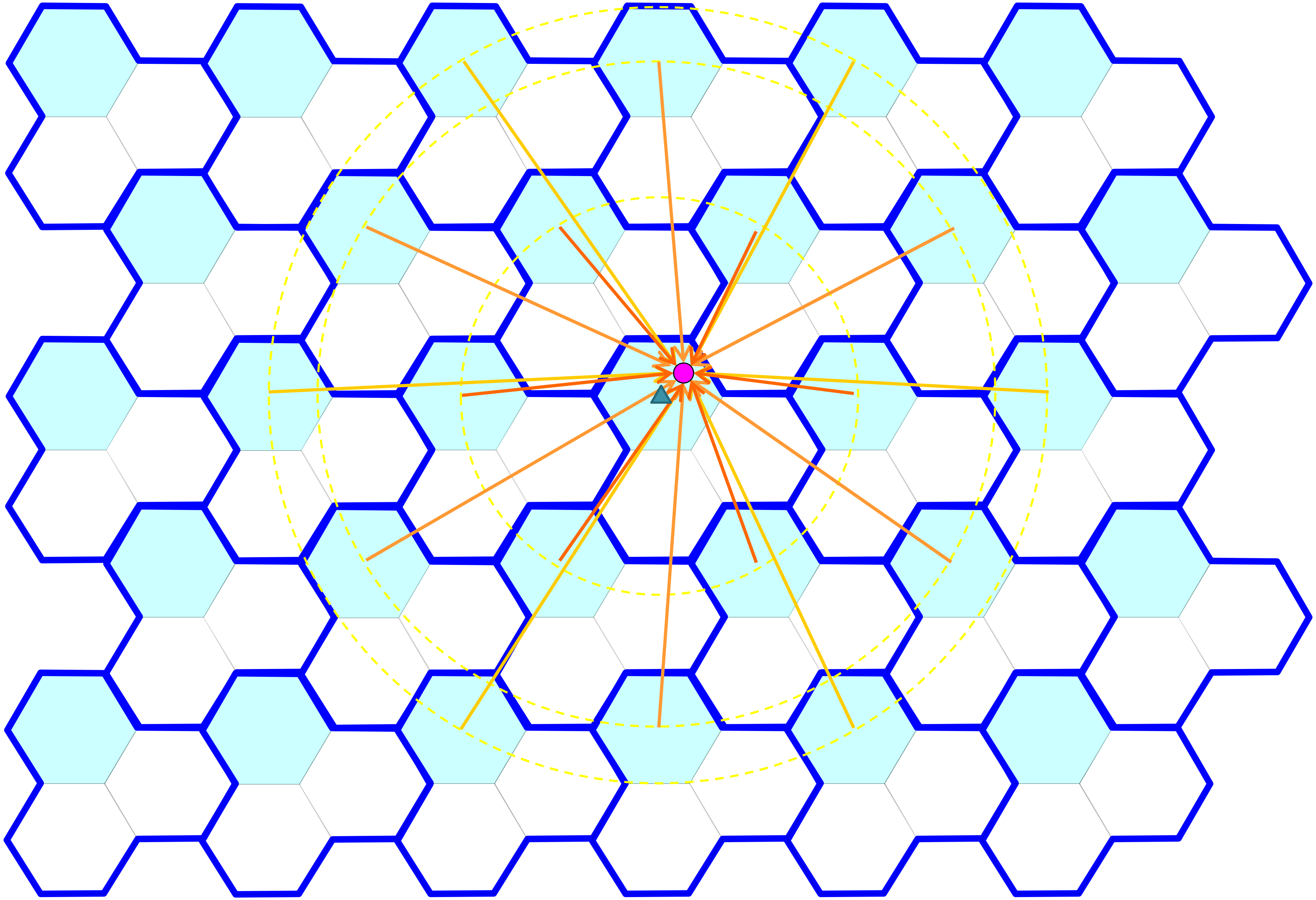
- K = # co-channel interfering cells
- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a forward channel can be expressed as

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

- P_r = the desired signal **power** from the desired base station
- P_i = the interference **power** caused by the *i*th interfering co-channel cell base station.
- Often called the **carrier-to-interference ratio**: CIR.

[Rappaport, 2002]

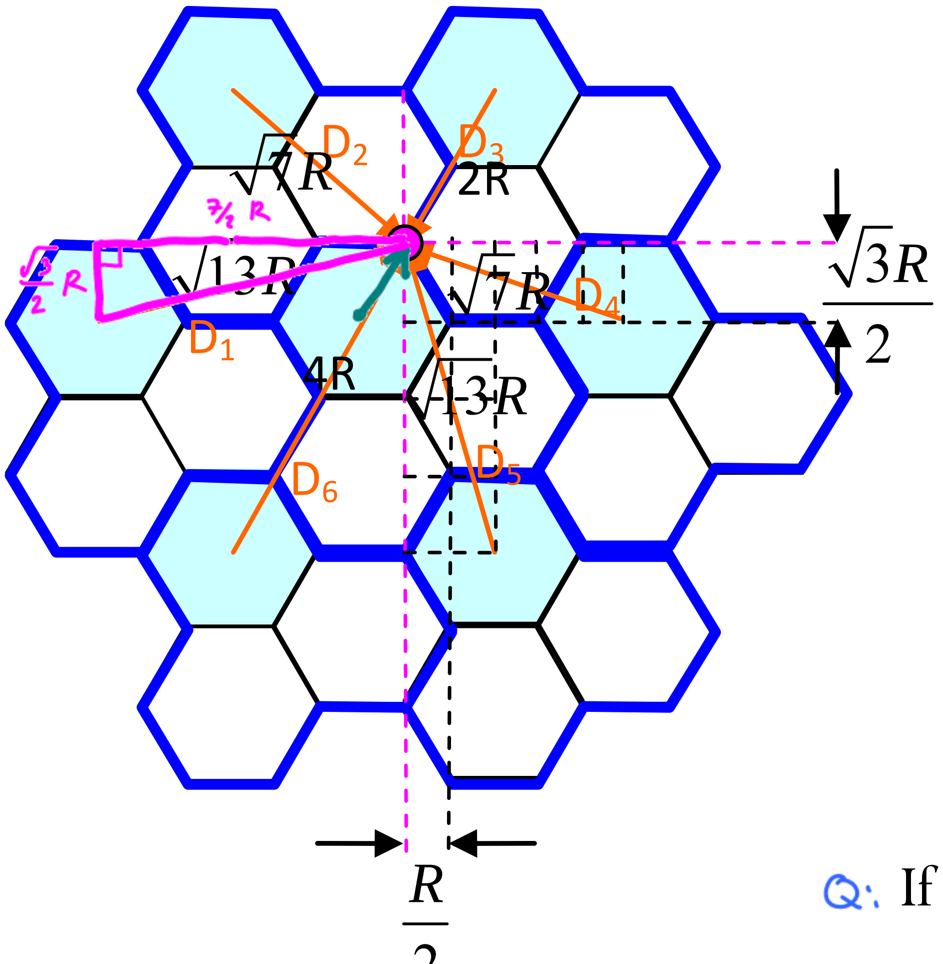
SIR: N = 3



SIR: N = 3

(Ignore co-channel cells that are too far away)

$D_1^2 = \left(\frac{7}{2}R\right)^2 + \left(\frac{\sqrt{3}}{2}R\right)^2 = R^2\left(\frac{49}{4} + \frac{3}{4}\right) = \frac{52}{4}R^2 = 13R^2$



- Consider only cells in first tier.
- Worse-case distance

P_r

$$\mathrm{SIR} \approx \frac{k/R^\gamma}{\sum_i k/D_i^\gamma} = \frac{1}{\sum_i 1 \Big/ \left(\frac{D_i}{R}\right)^\gamma} = \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

$\sum P_i$

$$= \frac{1}{2\left(\sqrt{7}\right)^{-\gamma} + 2\left(\sqrt{13}\right)^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}}$$

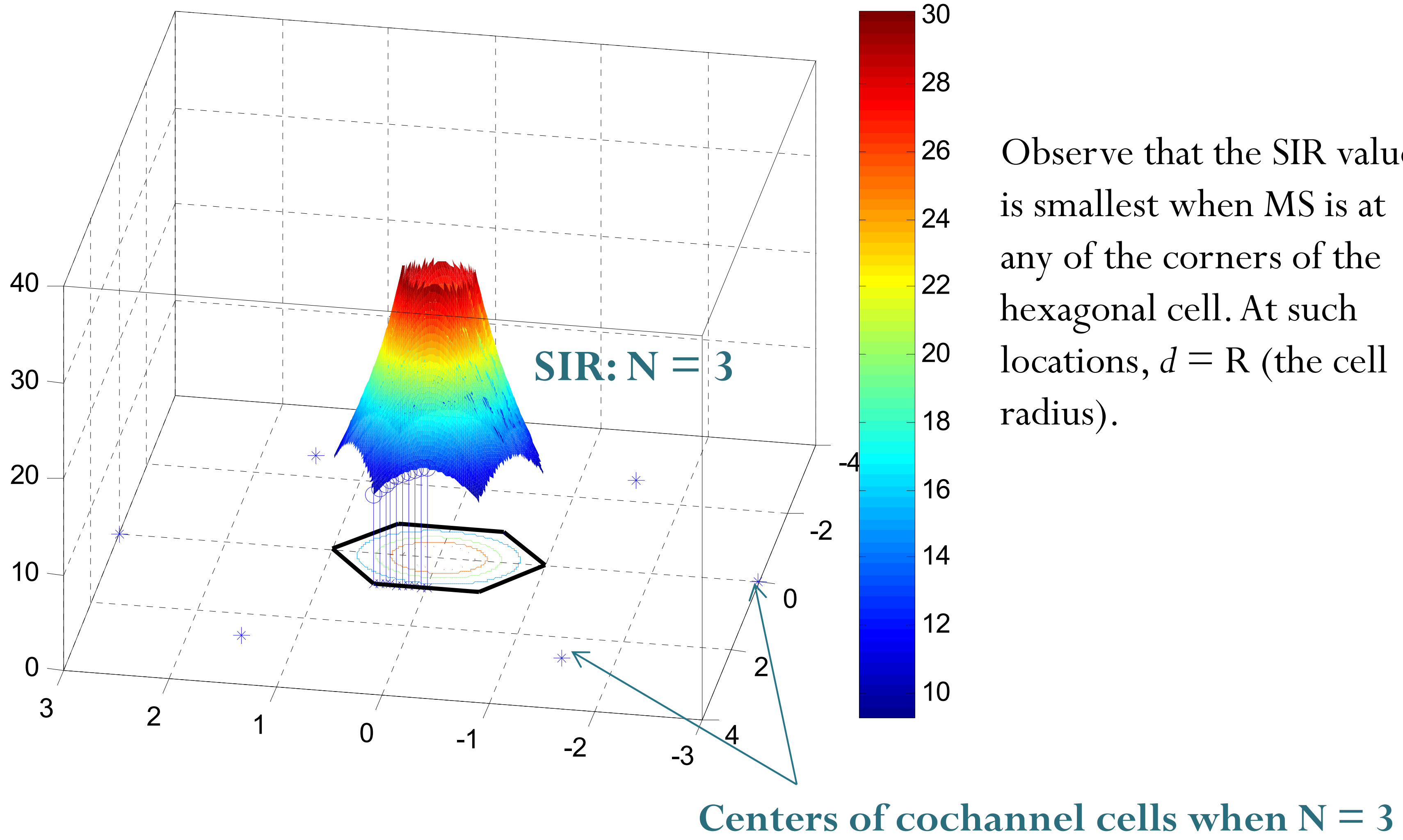
Q: If N = 19, will the SIR be better or worse?

A: Better ⇐ larger cluster size (greater distances among co-channel cells)

SIR: N = 3

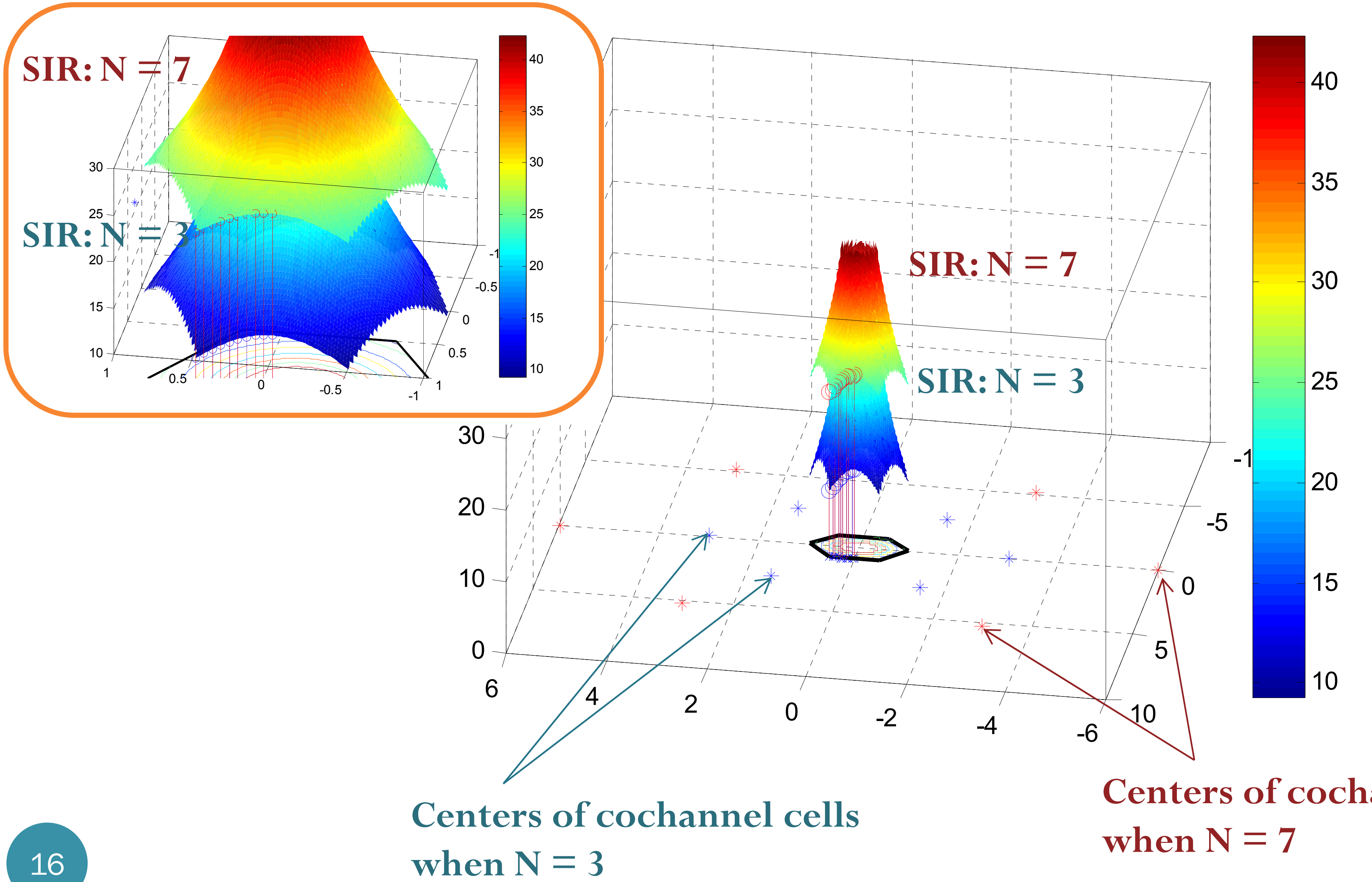
d = distance between MS and BS

$$\mathrm{SIR} \approx \frac{k/d^{\gamma}}{\sum_i k/D_i^{\gamma}} = \frac{1}{\sum_i 1\Big/\left(\frac{D_i}{d}\right)^{\gamma}} = \frac{1}{\sum_i \left(\frac{D_i}{d}\right)^{-\gamma}}$$



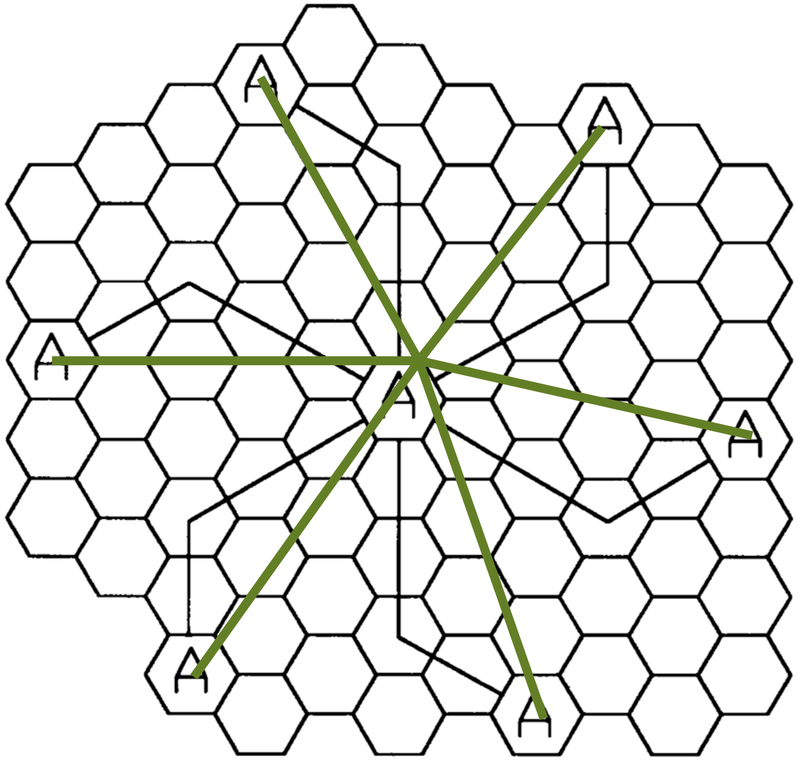
Observe that the SIR value is smallest when MS is at any of the corners of the hexagonal cell. At such locations, d = R (the cell radius).

SIR: N = 3 vs. N = 7



SIR: N = 7
SIR: N = 3
SIR: N = 7
SIR: N = 3
Centers of cochannel cells when N = 3
Centers of cochannel cells when N = 7

Approximation

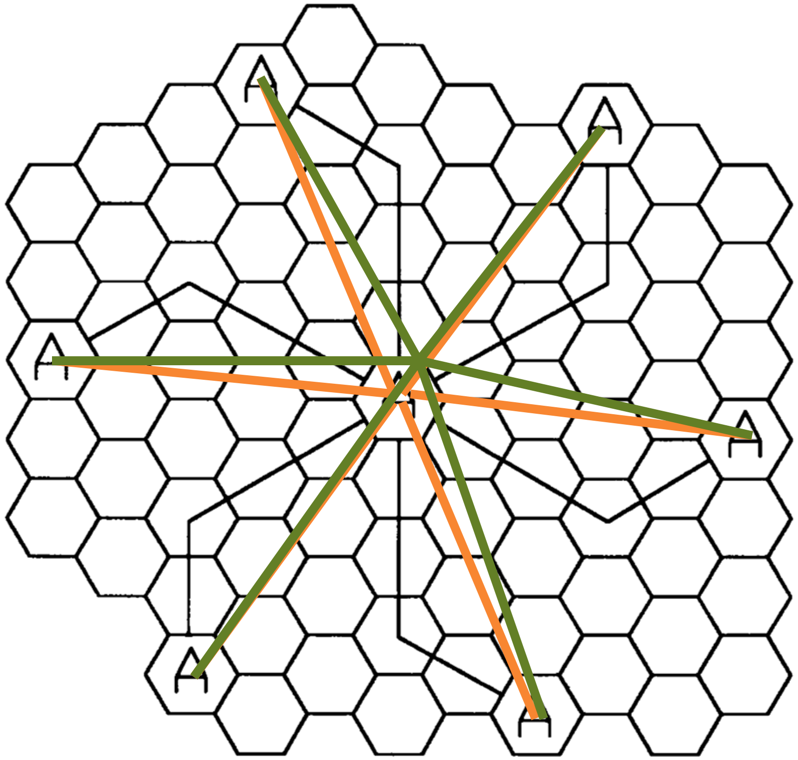


- Consider only first tier.
- Worse-case distance

$$\text{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

- Use the same D for D_i

Approximation



- Consider only first tier.
- Worse-case distance

$$\mathrm{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

- Use the same D for D_i

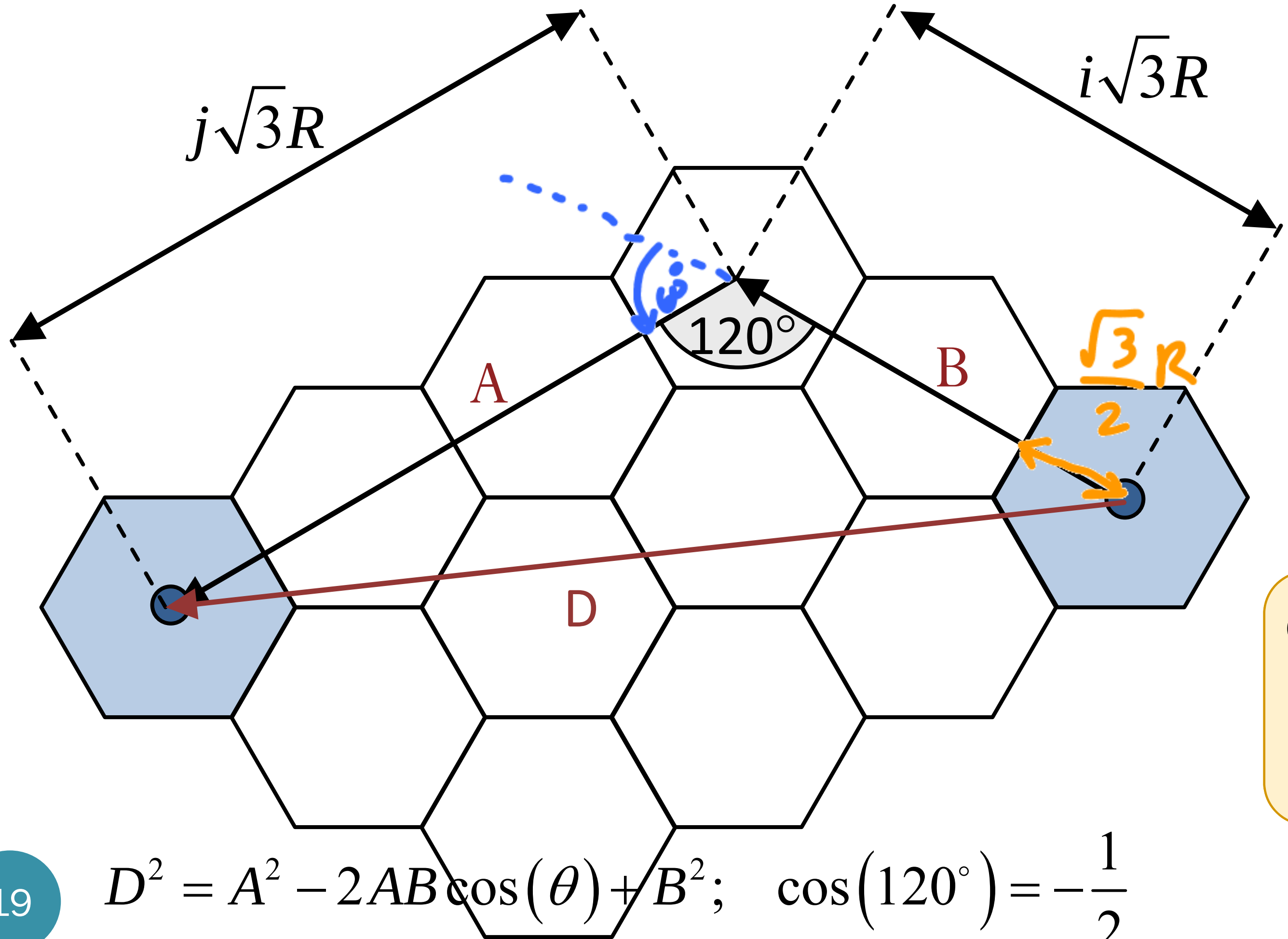
$$\mathrm{SIR} \approx \frac{1}{\sum_i \left(\frac{D}{R}\right)^{-\gamma}} \approx \frac{1}{K\left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma}$$

6

Notice that D/R is an important quantity!

Center-to-center distance (D)

$$D = \sqrt{\left(i\sqrt{3}R\right)^2 + \left(j\sqrt{3}R\right)^2 - 2\left(i\sqrt{3}R\right)\left(j\sqrt{3}R\right)\cos\left(120^\circ\right)}$$
$$= R\sqrt{3\left(i^2 + j^2 + ij\right)} = R\sqrt{3N}$$



This distance, D, is called **reuse distance**.

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

$$D^2 = A^2 - 2AB\cos(\theta) + B^2; \quad \cos\left(120^\circ\right) = -\frac{1}{2}$$

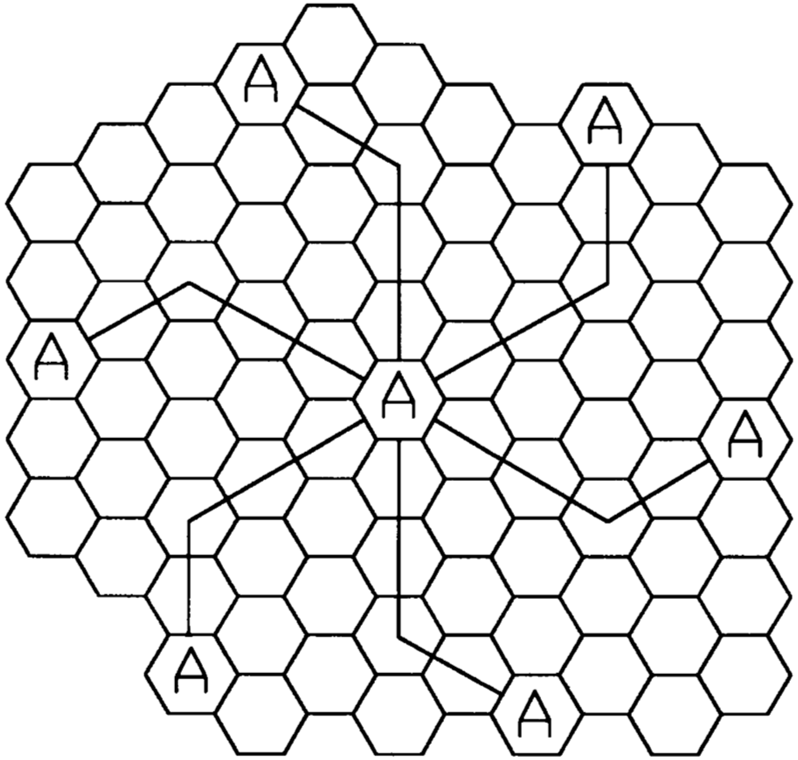
Q and N

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

	Cluster Size (*N*)	Co-channel Reuse Ratio (*Q*)
$i = 1, j = 1$	3	3
$i = 1, j = 2$	7	4.58
$i = 0, j = 3$	9	5.20
$i = 2, j = 2$	12	6

Approximation: Crude formula



$$\mathrm{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

$$\approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \approx \frac{1}{K\left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma}$$

$$= \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

$R\sqrt{3N}$

$= \frac{1}{K} 3N$

$\gamma = 2$

As the cell cluster size (N) increases, the spacing (D) between interfering cells increases, reducing the interference.

Summary: Quantity vs. Quality

S = total # available duplex radio channels for the system

"Capacity" $C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$

Tradeoff

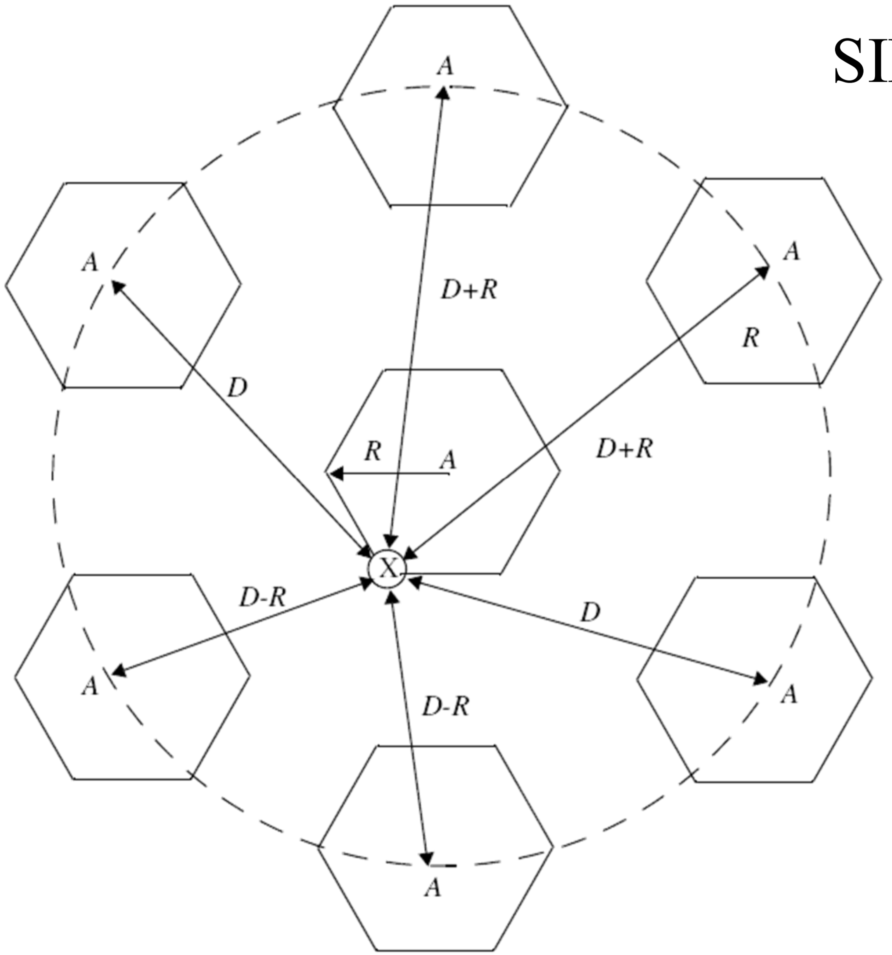
$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$

Path loss exponent

m = # channels allocated to each cell.

Frequency reuse with **cluster size *N***

SIR: N = 7



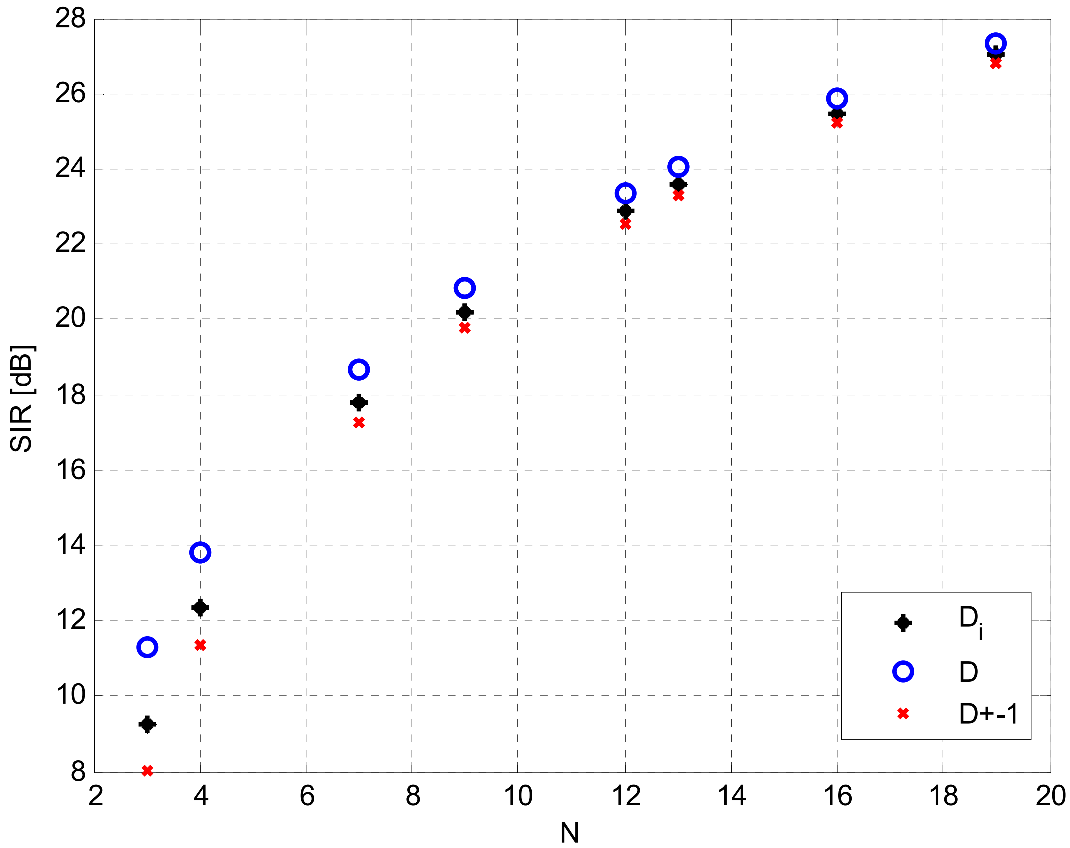
Better approximation…

$$\mathrm{SIR} \approx \frac{R^{-\gamma}}{2(D-R)^{-\gamma} + 2(D+R)^{-\gamma} + 2D^{-\gamma}}$$

$$= \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}$$

Again, $Q = \dfrac{D}{R} = \sqrt{3N}.$

Comparison



$$\mathrm{SIR} \approx \frac{1}{\sum_{i=1}^{6}\left(\frac{D_i}{R}\right)^{-\gamma}}$$

$$\mathrm{SIR} \approx \frac{1}{6}Q^{\gamma}$$

$$\mathrm{SIR} \approx \frac{1}{2(Q-1)^{-\gamma}+2(Q+1)^{-\gamma}+2Q^{-\gamma}}$$

$$Q = \frac{D}{R}$$

SIR Threshold

[Schwartz, 2005, p 64]

- The SIR should be greater than a specified threshold for proper signal operation.
- In the 1G **AMPS** system, designed for **voice** calls, the threshold for acceptable voice quality is SIR equal to **18 dB**.
- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.
- For the **GSM** system, a range of **7–12 dB**, depending on the study done, is suggested as the appropriate threshold.
- The probability of error in a digital system depends on the choice of this threshold as well.